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From Complex Problems to Simple Solutions: a Systematic Approach

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Abstract

A common problem familiar to many researchers dealing with complex technical systems (which can be formally described as non-stationary and/or non-linear multi-degree of freedom systems) is the need to find a meaningful solution which would have physical sense, would explicitly show dependence on the parameters and allow interpretation. Several decades ago the culture of building first approximation, asymptotical or slow time solutions was highly developed and practiced. Nowadays, with the advent of modern computers and numerical packages it often seems straightforward to generate a solution for the given set of parameters and boundary conditions. Therefore, the acuteness of this problem may be less obvious for the researcher. However, this "frontal attack" solution in some cases may be impractical (for instance, if this is an optimal control problem, the solution may require rapid changes of the control, which are hard to realize). In other cases, when the question arises as to what happens with the solution when the parameters change, the only answer may be to run the analysis again, which can be time consuming and still not show an interpretable dependency on the parameters.

Using a model solution can also help in optimization of a complex system, requiring multiple design iterations. The transition to a model solution in this case can be based on identifying a single characteristic or parameters of the system which has to meet contradictory requirements. While identification of such parameter may not be obvious, it can lead to resolving the contradiction for the model system using known problem solving tools (from the game theory to TRIZ). This solution needs then be mapped back to the initial system. The contradiction-solving model solution often offers a way to reach the goal of the project in a different way, obviating the need for the intensive numerical solution. The approach is illustrated by three case studies.

Keywords: closed-form solutions, contradictions, design optimization, model system.

1.Introduction

When dealing with complex technical systems (non-stationary and/or non-linear), it is often attractive to single out a simplified sub-system which carries most of the information needed for the researcher, and consider contribution of other variables or degrees of freedom as refining factors which do not substantially change the solution for the simplified sub-system. For the engineering systems having a small parameter, the perturbation methods have been used extensively and are reflected in numerous publications. In many applications, however, the initial

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(complex) system has no small parameter, and therefore the researcher needs to identify the simplified sub-system, based on his experience and intuition. Generally speaking, the researcher needs to identify in the initial system two sub-systems in such a way that the solution of the first (simplified) sub-system can be easily built and the solution of the second sub-system is small (in some sense) in the vicinity of the solution for the first system.

This approach is illustrated by three case studies, summarized in Table 1. The steps described in Table 1 can be described as follows.

Step 1. Restate the initial problem

The researcher must be able to transform the initial non-linear or/and non-stationary problem into a problem for a model system, which a) qualitatively has similar relationship between input and output variables and is based on the same principle of operation, and b) can be solved analytically, or which a solution is known. While this transformation needs to be selected on case by case basis, some general recommendations are:

- In a multi degree of freedom (DOF) system, single out a one DOF system that corresponds to the resonating natural mode;
- In a system with distributed parameters, seek solution in the form of series over natural modes of the system (eigen-function series), and then separate a sub-system having lower eigen-values (corresponding to slow variables);
- In a system with fast and slow variables, introduce averaging, and make a transition to a system in “slow time”, having only slow variables;
- In a system with time dependent variables, “freeze” those variables that change slowly and build a solution for the system with constant coefficients, et al.

It should be emphasized that the use of the procedures listed above can never be formal. The researched needs to deeply understand the problem in order to be able to single out a model sub-system. Applicability of the listed “recipes” needs to be validated in every case by applying the solution, built under the listed assumptions, to the initial system, or by direct experiments.

Step 2. Solve the Restated Problem

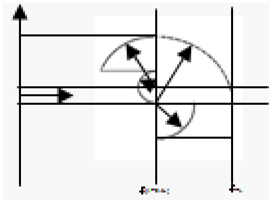
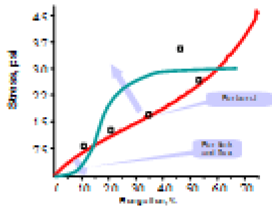
This is usually a relatively straight forward step. If Step 1 was done properly, the restated system should allow an analytical or known from a textbook solution, or in some cases relatively simple numerical solution.

Step 3. Apply Solution to the Initial Problem

Substitute the generated solution in the initial system, or, in case of an optimal control problem, apply the generated control variable to the initial system. Evaluate the inaccuracy or residuals. Depending on the achieved accuracy, steps 1 and 2 may need to repeat iteratively.

In what follows the case studies presented in Table 1 are discussed in more detail.

Table 1. Summary of the three case studies

	Case Study 1	Case Study 2	Case Study 3
			
Initial Problem	Optimal control problem for non-linear non-stationary system (acceleration of the rotor)	Analysis of non-stationary system with distributed parameters (fatigue problem for the cam-driven needles)	Optimization of design under contradictory requirements (Finite Element Analysis of high pressure catheter)
Step 1. Restate the Problem	Transform the initial (fast) variables into slow variables	Transform the initial system into superposition of one DOF (degree of freedom) systems	Identify in the initial multi-parameter system a single characteristic or parameter which has to meet contradictory requirements.
Step 2. Solve the Restated Problem	Solve the optimal control problem for slow variables (sub-optimal control)	Use model solution for one DOF system	Exacerbate and resolve the contradiction using known tools
Step 3. Apply Solution to the Initial Problem	Review response of the initial (fast) variables under the synthesized slowly changing control	Build solution for the initial system based on aggregation of model solutions	Map the solution back to the initial system

2.Optimization of the shape of the cam driven needles

The needles of the high-speed circular knitting machines often experience fatigue breakage of the needle head, due to high frequency vibration transmitted to the head from the driving point (the cam). The vibration is especially pronounced at a few frequencies of the spectrum, which are called response frequencies. The optimization goal in this case is to minimize the transfer functions (the ratio of the displacement at the driving point, which is the cam, to the displacement at the response point, which is the tip of the needle). The transfer functions are frequency dependent, and in this problem they need to be minimized at the response frequencies.

The known FEA packages can handle dynamics of a system with impulse loading as a general non-stationary problem, producing extensive output for each design iteration. However, these data will give no indication as to the direction for the required design change. Much more productive for the optimization process would be to use analytical solutions for a one DOF system under recurring impulses (δ-functions of amplitude A) at the moments *0, T, 2T, …*. The available software does not allow doing this directly. However, it is possible to determine from the (digitalized) stiffness and mass matrices of the distributed system, generated by the FEA packages, parameters of the equivalent one DOF systems that correspond to the natural modes of the needle, and response frequencies that provide maxima to the transfer functions from the driving point to the head of the needle. Only those natural modes that correspond to frequencies providing maxima to the transfer functions (from the driving point to the head of the needle) need to be selected. Analyzing the analytical solutions (unavailable in FEA) for each one DOF system makes it possible to identify the modes responsible for accumulation of damage at the tip and suppress these modes by design changes (Author, 1995).

In a more formal way, the solution process can be described as follows.

Resonances of the one DOF system with damping β and natural frequency p under periodically recurring impulses (δ-functions of amplitude A at the moments $0, t, 2t, \ldots$)

$$\ddot{x} + \beta\dot{x} + p^2 x = \sum_{n=-\infty}^{\infty} A\delta(t - n\tau), \tag{1}$$

where $\tau = 2\pi / \omega$ occur at frequencies $\omega = p / k, k = 1,2,...$

There are at least four possible ways to construct an analytical solution of (1); the most compact and computationally effective one was proposed by H. Duffing (Author, 1995). It is based on the condition of periodicity and has the form, in case of zero damping

$$x(t) = \frac{A}{2p}[\cot(p\tau/2)\cos(pt) + \sin(pt)], \quad 0 \le t \le \tau \tag{2}$$

The FEA model of the needle (Figure 1) is essentially a multi-DOF system, with the loading being a periodic function of time. It would be natural to generalize the approach which works well for a one DOF system to the multi-DOF case. In order to do that, the

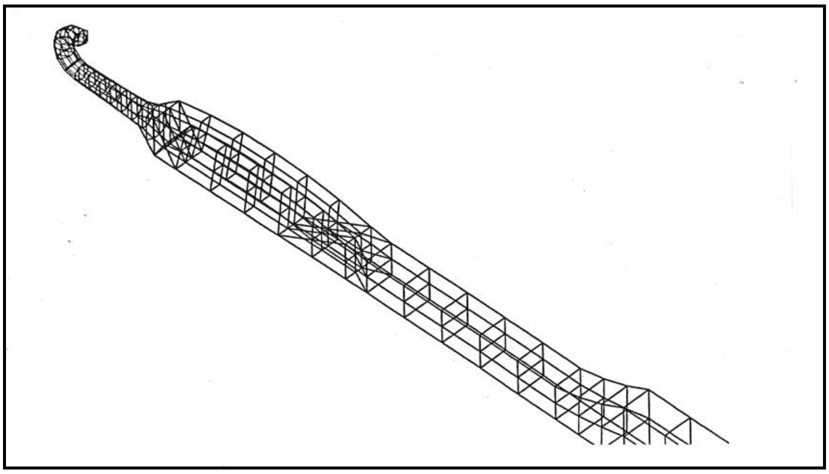


Figure 1. A FEA model of the needle of a high speed knitting machine

Following procedure was developed to estimate the fatigue life of the needle:

1. (a) construction of the direct analytical solution (2) for a one DOF system with damping and periodic non-harmonic excitation;
2. (b) modal analysis of the system by a FEA package (ALGOR, ANSYS, et al), in order to obtain parameters of the respective one DOF systems, corresponding to the response frequencies;
3. (c) superposition of the solutions for the response frequencies and summation of fatigue damage according to a selected hypothesis

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This procedure made it possible to obtain stresses in the hook of the needle for the baseline design and determine that the stress level was close to the endurance level. The analysis is illustrated by Figure 2, showing simulated stress history. It is important that only some specified harmonics need to be included in the stress estimates. Use of the analytical solution made it possible to identify those components of the needle that are mostly responsible for the transmission of the respective harmonics. The design of the needle was appropriately modified.

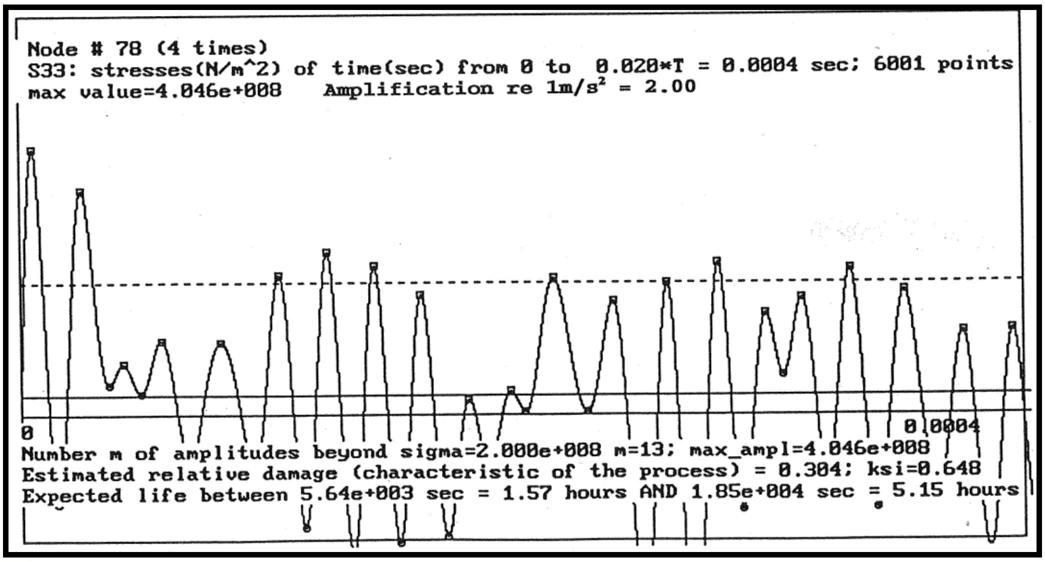


Figure 2. Simulated stress history in one of the node of the hook

3.Optimal control of acceleration of an imbalanced rotor through its critical speed

A more complicated situation arises in the optimization problem for an unbalanced rotor which needs to be sped up through its critical speed (a shaft, or rotor, rotates on a critical speed when rotation frequency of the shaft becomes close to its natural frequency, causing excessive vibrations of the rotary machine). If the operational speed of the rotor is higher than its critical speed, the process of acceleration of the rotor machine to its operational speed often becomes the most critical regime of the machine. In most cases, speeding up is done simply by turning the drive on, with no attempts to influence or control the process. Thus, the acceleration regime determines power requirements (the driving torque is increased in order to speed up the rotor and shorten the time required to pass the critical speed), level of vibration and other major parameters of the rotating machine.

An estimate of the minimum driving torque u_{min} required to speed up the rotor through the critical speed is known from literature (Gasch et al, 1979). This estimate is obtained under the assumption that the torque is constantly on over the time of acceleration. However, once the driving torque is considered as an available control influence, then the optimization goal can be stated as to minimize power of the drive (or the maximum torque) which is capable to accelerate the rotor above its critical speed.

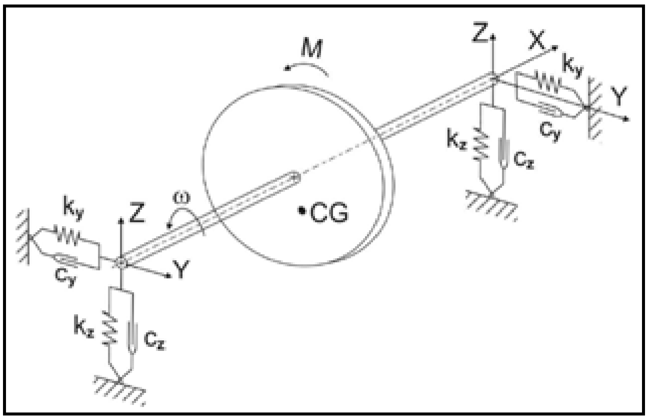


Figure 3. Three DOF Laval rotor. X, Y, Z – coordinate system, c_y, c_z – damping coefficients, k_y, k_z – stiffness coefficients, ω – angular velocity, M – torque, CG – center of gravity

The optimization problem, if based on the initial non-stationary non-linear dynamic equations which describe acceleration of the rotor, is insurmountable for the available numerical algorithms, even for the 3 degrees of freedom Laval rotor (Figure 3) , represented by Equations (3).

However, within the framework of the proposed approach, this problem can be addressed in a sequence of the following steps (Author, 1992):

(a) Make a transition from the initial fast variables in the dynamic Equations (3) to slow variables in Equations (4).

$$\ddot{Z} + 2D\dot{Z} + Z = \cos\varphi,$$

$$\ddot{Y} + 2D\dot{Y} + Y = \sin\varphi, \qquad (3)$$

$$\ddot{\varphi} = u + \mu^2(Y\cos\varphi - Z\sin\varphi),$$

$$\dot{v} = -(1-\omega)w - Dv,$$

$$\dot{w} = -1/2 + (1-\omega)v - Dw, \qquad (4)$$

$$\dot{\omega} = u + \mu^2 w,$$

where $v = A\cos\delta,\ w = A\sin\delta,\ A = \sqrt{Z^2 + Y^2},\ \tan(\varphi - \delta) = Y / Z$.

In Equations (3), (4) differentiation takes place with respect to dimensionless time $\tau = \Omega t$; $\Omega = \sqrt{k/m}$; k is the stiffness of the shaft; m is the mass of the disk; $Z = z_s/\varepsilon$; $Y = y_s/\varepsilon$; ε is the eccentricity, z_s, y_s are the coordinates and φ is the angular coordinate of the disk's center of mass in a non-moving *z, y* coordinate system; $D = r/2m\Omega$; r is the external damping coefficient; $u = M/m\kappa^2\Omega^2$; $m = \varepsilon/\kappa$, M is the drive torque, κ is the radius of inertia of the disk.

(b) "Freeze" one of the slow changing variables in the obtained system (4). One can see that the derivative v' is proportional to a small parameter (in the vicinity of the critical speed, 1-ω is small, and damping *D* is also small). This will result in a linear system (5) for every value of the frozen variable v.

$$\dot{w} = -1/2 + (1-\omega)v - Dw,$$
$$\dot{\omega} = u + \mu^2 w \qquad (5)$$

(c) Build an optimal feedback-based solution for thus obtained linear system (with respect to the variables *w*, ω). To that end, we shall abandon the assumption that the torque *u* (or, *M*) is constant, and attempt to find a law of variation $u = u(t)$ that ensures that the rotor will reach an above-critical speed ($\omega(T) = \omega_T > 1$) at a time T and minimizes a certain functional J (quality criterion) with limitations on the drive torque:

$$u_- \le u \le u_+$$

with the focus on the case when $u_+ < u_{min}$, and $u_{min} = 1.3\mu^{4/3}$ is the estimate (for D=0) for the minimum dimensionless constant torque u = const necessary to pass through the critical speed ($\omega_c = 1$) (Gasch et al, 1979).

(d) Apply the solution to the initial non-linear non-stationary system (3) to confirm its workability.

For a real rotor machine, and extra step prior to step (a) would be to diagonalize the system, presenting it as a set of sub-systems each described by Equations (3) for the respective critical frequencies (similar to how it was done in the previous case study for the knitting machines).

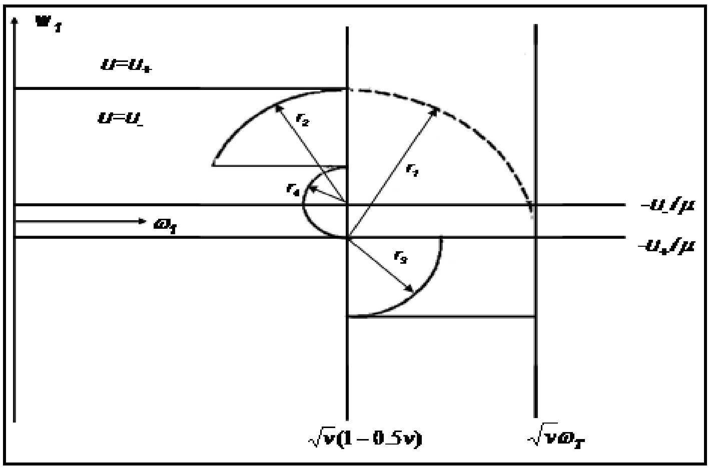


Figure 4. Switching lines in the phase plane of slow variables (ω, *w*)

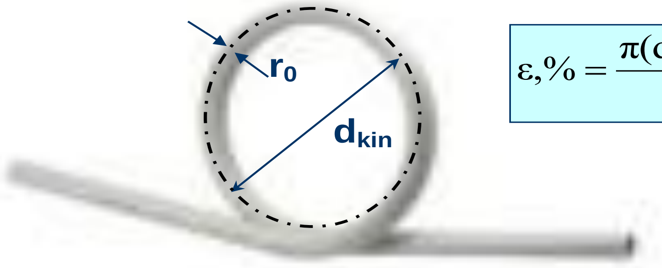
This approach makes it possible to synthesize a feedback-based solution, which can be then applied to the initial (fast) system. The optimal control is of relay (bang-bang) nature, with the driving torque u taking in turn maximum u_{max} and minimum u_{min} values (the drive is on and off). The switching lines in the phase plane of the slow variables (w, ω) are shown in Figure 4. Extensive modeling and experiments (Author, 1992) have confirmed efficiency of such a control system with feedback of a measurable phase coordinate, which ensures acceleration of the rotor at greatly reduced drive torque.

1. Material optimization

Catheters are routinely used to transfer fluids into the body without repeatedly inserting a needle through the skin. In many cases, the catheter must be able to operate in multiple modes, which can present contradictory requirements to the design and material of the catheter. For instance, a peripherally inserted central catheter (PICC) must be able to hold sufficiently high pressure and at the same time be highly flexible to withstand the so-called kink tests.

The first requirement is stipulated by the regime when fluids, which are infused through the catheter, are supplied from a pressurized source. The speed of infusion is important, as faster infusion reduces the time to administer a treatment and the cost of the procedure. Infusing under pressure demands sufficient strength of the catheter.

The second requirement, the kink tests (Figure 5) and related high elasticity of the catheter, reflect operational conditions when the catheter can be folded many times at the arm of the patient.



$$\varepsilon,\% = \frac{\pi(d_{kink} + r_0) - \pi d_{kink}}{\pi(d_{kink} + r_0)} 100$$

Figure 5. Kink deformation of the catheter

Table 2. High elasticity for the kink test is only needed at low deformations

6-4F	Tip		Body	
d_{kink}, mm	*r_0, mm*	*ε, %*	*r_0, mm*	*ε, %*
6.985	0.673	9	1.00	13
15.24	0.673	4	1.00	6

This leads to contradictory requirements to the design of the catheter, which are usually addressed through multiple material and design iterations that represent a trade off between the two contradictory requirements. However, this problem can be recast as identification of a catheter material with contradictory properties, high elasticity (for the kink tests) and at the same time high strength (for the burst tests). This boils down to identification of a material which meets contradictory requirements to a single characteristic, its stress-deformation curve. The contradiction can be resolved based on the realization that high elasticity (the kink tests) is required at low deformations (Table 2) and high strength (burst tests) at large deformations, therefore, the requirements can be separated in *the space of elastic parameters* of the material (Figure 6). The desired (non-linear) stress-elongation characteristic would represent very elastic material at low deformations, toughening up as the deformations grow. The material with the desired characteristic can be indeed designed, as shown in (Bell et al, 2008; DiCarlo et al, 2007).

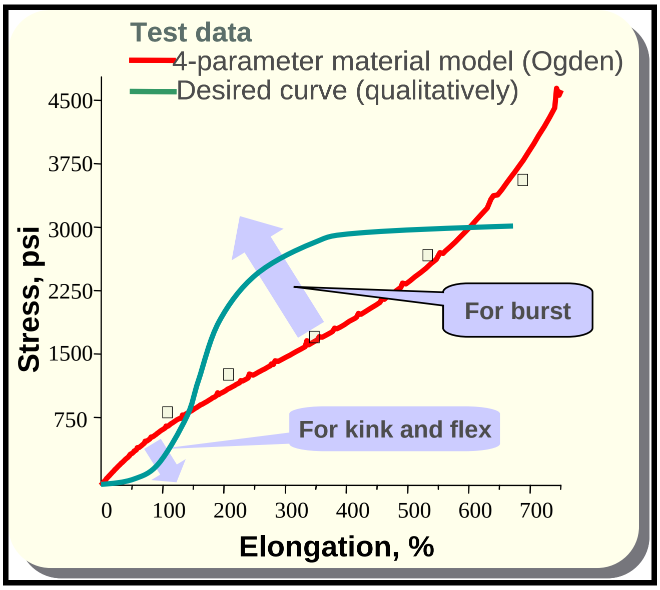


Figure 6. Elastic characteristic of the catheter material: existing material (red curve, 4 parameter Ogden model) and proposed material (green curve)

2. Conclusions

The approach outlined in this article can be summarized as follows: when dealing with a complex engineering problem, construct a simplified subset or sub-system of the initial system having the main features of the initial system but for which an analytical closed form solution can be built or is known. Study how the model solution depends on the parameters of the constructed sub-system. Generalize or back propagate the model solution to the initial system. Conduct computer modeling or direct experiment to validate the solution.

This approach is illustrated by three case studies: optimization of a needle shape, optimal control of rotor acceleration, optimization of material properties of a catheter.

3. Acknowledgements

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