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Brand Selection Model with the Expansion to the Second Order Lag

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Abstract

By focusing the following condition that consumers' are apt to buy superior brand when they are accustomed or bored to use current brand, a new analysis method is introduced. The data set of "before buying data" and "after buying data" is stated using the liner model. When above stated events occur, transition matrix becomes upper triangular matrix. In this paper, equation using transition matrix is extended to the second order lag and the method is newly re-built in order to improve forecasting accuracy. These are confirmed by the numerical example. S-step forecasting model is also introduced. This approach makes it possible to identify brand position in the market and it can be utilized for building useful and effective marketing plan.

Keywords: brand selection, matrix structure, brand position

1. Introduction

It is often observed that consumers select the upper class brand when they buy the next time. Focusing the transition matrix structure of brand selection, their activities may be analyzed. In the past, there are many researches about brand selection (Aker, 1991; Katahira, 1987; Katahira & Sugita, 1994; Takahashi & Takahashi, 2002; Yamanaka, 1982). But there are few papers concerning the analysis of the transition matrix structure of brand selection. In this paper, we make analysis of the preference shift of customer brand selection and confirm them by the numerical example. If we can identify the feature of the matrix structure of brand selection, it can be utilized for the marketing strategy.

Suppose that the former buying data and the current buying data are gathered. Also suppose that the upper brand is located upper in the variable array. Then the transition matrix becomes an upper triangular matrix under the supposition that the former buying variables are set input and the current buying variables are set output. If the top brand were selected from the lower brand in jumping way, corresponding part in the upper triangular matrix would be 0. These are verified by the numerical examples with simple models.

If the transition matrix is identified, a S-step forecasting can be executed. Generalized forecasting matrix components' equations are introduced. Unless planner for products does not notice its brand position whether it is upper or lower than another products, matrix structure make it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand.

A quantitative analysis concerning brand selection has been executed in prior researches (Takahashi & Takahashi, 2002; Yamanaka, 1982. Yamanaka (1982) examined purchasing process by Markov Transition Probability with the input of advertising expense. Takahashi and Takahashi (2002) made analysis by the Brand Selection Probability model using logistics distribution. Takeyasu and Higuchi (2007) suggested that matrix structure was analyzed for the case brand selection executed toward upper class.

In this paper, equation using transition matrix is extended to the second order lag and the method is newly re-built in order to improve forecasting accuracy. Such research is quite a new one.

Hereinafter, extended analysis method is stated in section 2. Matrix structure is clarified for the brand selection in section 3. Extension of the model to the second order lag is executed in section 4. Forecasting is formulated in section 5. Numerical calculation is executed in section 6. Section 7 is a summary.

2. Extended Analysis Method

Zlotin and Zusman (2006) proposed the concept of "Trends" in TRIZ CON 2006. We can further develop this concept as shown in Figure 1.

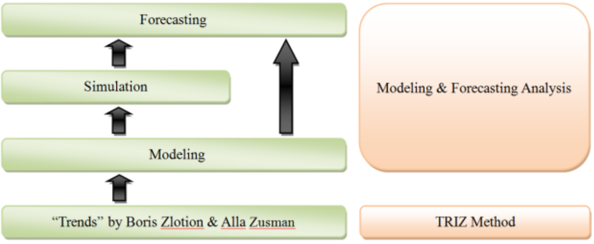

Fig. 1. Extended Analysis Method

Based on the TRIZ method, modeling and forecasting analysis method is developed. Extending "Trends", modeling is constructed first. Then we can make simulation by utilizing them. We can make forecasting utilizing the simulation function or directly from the utilization of the model built. These are the process of "Modeling & Forecasting Analysis" based upon TRIZ "Trends" analysis method. In this paper, the problem is to improve forecasting accuracy. The way to solve or cope with this problem is exhibited in Figure 2. Method is taken by building the expansion to the second order lag model in order to improve forecasting accuracy to the objective value. Detailed inspection is executed in Section 5 through 7.

<Problem & Issues>
Forecasting accuracy should be improved

<Aim>
Improve forecasting accuracy

<Method>
Model Expansion

<Detailed Method>
Expansion to the Second Order Lag Model

Fig. 2. The way to solve or cope with the problem

3. Brand Selection and its Matrix Structure

3.1 Upper Shift of Brand Selection

It is often observed that consumers select the upper class brand when they buy the next time. Now, suppose that x is the most upper class brand, y is the second upper brand, and z is the lowest brand. Consumer's behavior of selecting brand would be $z \to y$, $y \to x$, $z \to x$ etc. $x \to z$ might be few.

Suppose that x is the current buying variable, and x_b is the previous buying variable. Shift to x is executed from x_b, y_b, or z_b. Therefore, x is stated in the following equation.

$$x = a_{11}x_b + a_{12}y_b + a_{13}z_b$$

Similarly, $y = a_{22}y_b + a_{23}z_b$

And $z = a_{33}z_b$

These are re-written as follows.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} \quad (1)$$

Set:

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}, \quad \mathbf{X_b} = \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

Then, $\mathbf{X}$ is represented as follows.

$$\mathbf{X} = \mathbf{AX_b} \quad (2)$$

Here,

$$\mathbf{X} \in \mathbf{R}^3, \mathbf{A} \in \mathbf{R}^{3\times 3}, \mathbf{X_b} \in \mathbf{R}^3$$

$\mathbf{A}$ is an upper triangular matrix. To examine this, generating the following data, which are all consisted by the upper brand shift data.

$$\mathbf{X^i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \cdots, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad (3)$$

$$\mathbf{X_b^i} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \cdots, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (4)$$

$$i = 1, \quad 2, \quad \cdots, \quad N$$

Parameter can be estimated using least square method. Suppose

$$\mathbf{X}^i = \mathbf{A}\mathbf{X}_\mathbf{b}^i + \boldsymbol{\varepsilon}^i \qquad (5)$$

Where

$$\varepsilon^i = \begin{pmatrix} \varepsilon_1^i \\ \varepsilon_2^i \\ \varepsilon_3^i \end{pmatrix} \qquad i = 1,2,\cdots,N$$

And

$$J = \sum_{i=1}^{N} \boldsymbol{\varepsilon}^{iT}\boldsymbol{\varepsilon}^i \rightarrow Min \qquad (6)$$

$\hat{\mathbf{A}}$ which is an estimated value of $\mathbf{A}$ is obtained as follows.

$$\hat{\mathbf{A}} = \left(\sum_{i=1}^{N} \mathbf{X}^i \mathbf{X}_\mathbf{b}^{iT}\right)\left(\sum_{i=1}^{N} \mathbf{X}_\mathbf{b}^i \mathbf{X}_\mathbf{b}^{iT}\right)^{-1} \qquad (7)$$

In the data group of the upper shift brand, estimated value $\hat{\mathbf{A}}$ should be an upper triangular matrix. If the following data, that have the lower shift brand, are added only a few in equation (3) and (4),

$$\mathbf{X}^i = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_\mathbf{b}^i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\hat{\mathbf{A}}$ would contain minute items in the lower part triangle.

If $\mathbf{X}_\mathbf{b}$ is replaced by $\mathbf{X}$ in the right hand side of Eq.(2) and by utilizing $\hat{\mathbf{A}}$, forecasting can be executed by Eq.(2) (The value of the left hand side becomes the forecasting of $\mathbf{X}$).

3.2 Sorting Brand Ranking by Re-arranging Row

In a general data, variables may not be in order as x, y, z. In that case, large and small values lie scattered in $\hat{\mathbf{A}}$. But re-arranging this, we can set in order by shifting row. The large value parts are gathered in an upper triangular matrix, and the small value parts are gathered in a lower triangular matrix.

$$\begin{array}{cc} & \hat{\mathbf{A}} \\ \begin{pmatrix} x \\ y \\ z \end{pmatrix} & \begin{pmatrix} \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \\ \varepsilon & \varepsilon & \bigcirc \end{pmatrix} \end{array} \underset{\Longleftarrow}{\text{Shifting}} \begin{array}{cc} & \hat{\mathbf{A}} \\ \begin{pmatrix} z \\ x \\ y \end{pmatrix} & \begin{pmatrix} \varepsilon & \varepsilon & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \end{pmatrix} \end{array} \qquad (8)$$

3.3 Matrix Structure under the Case Intermediate

Class Brand is Skipped

It is often observed that some consumers select the most upper class brand from the most lower class brand and skip selecting the middle class brand. We suppose v, w, x, y, z brands (suppose they are laid from the upper position to the lower position as $v > w > x > y > z$). In the above case, the selection shifts would be

$$v \leftarrow z, \quad v \leftarrow y$$

Suppose there is no shift from z to y, corresponding part of the transition matrix is 0 (i.e. a_{45}=0). Similarly, if there is no shift from z to y, from z to w, from y to x, form y to w, from x to w, then the matrix structure would be as follows.

$$\begin{pmatrix} v \\ w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 \\ 0 & 0 & 0 & 0 & a_{55} \end{pmatrix} \begin{pmatrix} v_b \\ w_b \\ x_b \\ y_b \\ z_b \end{pmatrix} \qquad (9)$$

4. Expansion of the Model to the Second Order Lag

We extend Eq.(2) to the second order lag in this section in order to improve forecasting accuracy. We have analyzed the automobile purchasing case (Takeyasu & Higuchi, 2007). In that case, we could obtain the data (current buying data, former buying data, before former buying data). We have analyzed them by dividing the data (current buying data, former buying data) and (former buying data before former buying data), and put them to Eq.(5) to apply the model.

But this is a kind of a simplified method to apply to the model. If we have a further time lag model and we can utilized the data as it is, the estimation accuracy of parameter would be accurate and the forecasting would be more precise. Therefore we introduce a new model which extends Eq.(2) to the second order lag model as follows.

$$\mathbf{X}_t = \mathbf{A}_1\mathbf{X}_{t-1} + \mathbf{A}_2\mathbf{X}_{t-2} \tag{10}$$

Where

$$\mathbf{X}_t = \begin{pmatrix} x_1^t \\ x_2^t \\ \vdots \\ x_p^t \end{pmatrix} \qquad t = 1,2\cdots$$

$$\mathbf{A}_1 = \begin{pmatrix} a_{11}^{(1)}, & a_{12}^{(1)}, & \cdots, & a_{1p}^{(1)} \\ a_{21}^{(1)}, & a_{22}^{(1)}, & \cdots, & a_{2p}^{(1)} \\ \vdots & \vdots & & \vdots \\ a_{p1}^{(1)}, & a_{p1}^{(1)}, & \ldots, & a_{pp}^{(1)} \end{pmatrix}$$

$$\mathbf{A}_2 = \begin{pmatrix} a_{11}^{(2)}, & a_{12}^{(2)}, & \cdots, & a_{1p}^{(2)} \\ a_{21}^{(2)}, & a_{22}^{(2)}, & \cdots, & a_{2p}^{(2)} \\ \vdots & \vdots & & \vdots \\ a_{p1}^{(2)}, & a_{p1}^{(2)}, & \ldots, & a_{pp}^{(2)} \end{pmatrix}$$

$$\mathbf{X}_t \in \mathbf{R}^p \ (t = 1,2,\cdots) \quad \mathbf{A}_1 \in \mathbf{R}^{p\times p}, \mathbf{A}_2 \in \mathbf{R}^{p\times p}$$

In order to estimate $\mathbf{A_1}, \mathbf{A_2}$, we set the following equation in the same way as before.

$$\mathbf{X}_t^i = \mathbf{A}_1\mathbf{X}_{t-1}^i + \mathbf{A}_2\mathbf{X}_{t-2}^i + \boldsymbol{\varepsilon}_t^i \quad (t = 1,2,\cdots,N) \tag{11}$$

$$J = \sum_{i=1}^{N} \boldsymbol{\varepsilon}_t{}^{iT}\boldsymbol{\varepsilon}_t{}^i \to Min \tag{12}$$

Eq.(11) is expressed as follows.

$$\mathbf{X}_t^i = \left(\mathbf{A}_1, \quad \mathbf{A}_2\right)\begin{pmatrix} \mathbf{X}_{t-1}^i \\ \mathbf{X}_{t-2}^i \end{pmatrix} + \boldsymbol{\varepsilon}_t^i \tag{13}$$

$\left(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2\right)$ which is an estimated value of $\left(\mathbf{A}_1, \quad \mathbf{A}_2\right)$ is obtained as follows in the same way as Eq.(7).

$$\left(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2\right) = \left(\sum_{i=1}^{N} \mathbf{X}_t^i \left(\mathbf{X}_{t-1}^{iT}, \quad \mathbf{X}_{t-2}^{iT}\right)\right)\left(\sum_{i=1}^{N} \begin{pmatrix} \mathbf{X}_{t-1}^i \\ \mathbf{X}_{t-2}^i \end{pmatrix}\left(\mathbf{X}_{t-1}^{iT}, \quad \mathbf{X}_{t-2}^{iT}\right)\right)^{-1} \tag{14}$$

This is re-written as :

$$\left(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2\right) = \left(\sum_{i=1}^{N} \mathbf{X}_t^i\mathbf{X}_{t-1}^{iT}, \quad \sum_{i=1}^{N} \mathbf{X}_t^i\mathbf{X}_{t-2}^{iT}\right)\begin{pmatrix} \sum_{i=1}^{N} \mathbf{X}_{t-1}^i\mathbf{X}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{X}_{t-1}^i\mathbf{X}_{t-2}^{iT} \\ \sum_{i=1}^{N} \mathbf{X}_{t-2}^i\mathbf{X}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{X}_{t-2}^i\mathbf{X}_{t-2}^{iT} \end{pmatrix}^{-1} \tag{15}$$

We set this as :

$$\left(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2\right) = \left(\hat{\mathbf{B}}, \quad \hat{\mathbf{C}}\right)\begin{pmatrix} \hat{\mathbf{D}} & \hat{\mathbf{E}} \\ \hat{\mathbf{E}}^T & \hat{\mathbf{F}} \end{pmatrix}^{-1} \tag{16}$$

In the data group of upper shift brand, $\hat{\mathbf{E}}$ becomes an upper triangular matrix. While $\hat{\mathbf{D}}$ and $\hat{\mathbf{F}}$
are diagonal matrix in any case. This will be made clear in the numerical calculation later.

5. Forecasting

After transition matrix is estimated, we can make forecasting. We show some of them in the following equations.

$$\hat{\mathbf{X}}_{t+1} = \hat{\mathbf{A}}_1\mathbf{X}_t + \hat{\mathbf{A}}_2\mathbf{X}_{t-1} \tag{17}$$

$$\hat{\mathbf{X}}_{t+2} = \left(\hat{\mathbf{A}}_1{}^2 + \hat{\mathbf{A}}_2\right)\mathbf{X}_t + \hat{\mathbf{A}}_1\hat{\mathbf{A}}_2\mathbf{X}_{t-1} \tag{18}$$

$$\hat{\mathbf{X}}_{t+3} = \left(\hat{\mathbf{A}}_1{}^3 + \hat{\mathbf{A}}_1\hat{\mathbf{A}}_2 + \hat{\mathbf{A}}_2\hat{\mathbf{A}}_1\right)\mathbf{X}_t + \left(\hat{\mathbf{A}}_1{}^2 + \hat{\mathbf{A}}_2\right)\hat{\mathbf{A}}_2\mathbf{X}_{t-1} \tag{19}$$

$$\hat{\mathbf{X}}_{t+4} = \left(\hat{\mathbf{A}}_1{}^4 + \hat{\mathbf{A}}_1{}^2\hat{\mathbf{A}}_2 + \hat{\mathbf{A}}_1\hat{\mathbf{A}}_2\hat{\mathbf{A}}_1 + \hat{\mathbf{A}}_2\hat{\mathbf{A}}_1{}^2 + \hat{\mathbf{A}}_2{}^2\right)\mathbf{X}_t + \left\{\hat{\mathbf{A}}_1\left(\hat{\mathbf{A}}_1{}^2 + \hat{\mathbf{A}}_2\right) + \hat{\mathbf{A}}_2\hat{\mathbf{A}}_1\right\}\hat{\mathbf{A}}_2\mathbf{X}_{t-1} \tag{20}$$

6. Numerical Example

In this section, we consider the case there is no shift to lower brand. We consider the case that brand selection shifts to the same class or upper classes. As above referenced, corresponding part of transition matrix must be an upper triangular matrix. Suppose following events occur. Here we set $p = 3$ in Eq.(10).

	$<\mathbf{X}_{t-2}$ to $\mathbf{X}_{t-1}>$		$<\mathbf{X}_{t-1}$ to $\mathbf{X}_t>$	
①	Shift from lower brand to middle brand	4events	Shift from middle brand to upper brand	4events
②	Shift from lower brand to lower brand	2events	Shift from lower brand to upper brand	2events
③	Shift from lower brand to lower brand	5events	Shift from lower brand to middle brand	5events
④	Shift from lower brand to upper brand	1events	Shift from upper brand to upper brand	1events

⑤	Shift from lower brand to middle brand	2events	Shift from middle brand to middle brand	2events
⑥	Shift from lower brand to lower brand	3events	Shift from lower brand to lower brand	3events
⑦	Shift from middle brand to middle brand	3events	Shift from middle brand to upper brand	3events
⑧	Shift from middle brand to middle brand	2events	Shift from middle brand to middle brand	2events
⑨	Shift from upper brand to upper brand	4event	Shift from upper brand to upper brand	4event
⑩	Shift from middle brand to upper brand	2events	Shift from upper brand to upper brand	2events
⑪	-		Shift from lower brand to middle brand	3events
⑫	-		Shift from middle brand to middle brand	2event
⑬	-		Shift from middle brand to upper brand	4events

Vector $\mathbf{X}_t, \mathbf{X}_{t-1}, \mathbf{X}_{t-2}$ in these cases are expressed as follows.

① $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

② $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

③ $\mathbf{X}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

④ $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

⑤ $\mathbf{X}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

⑥ $\mathbf{X}_t = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

⑦ $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

⑧ $\mathbf{X}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

⑨ $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

⑩ $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

⑪ $\mathbf{X}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

⑫ $\mathbf{X}_t = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

⑬ $\mathbf{X}_t = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{X}_{t-1} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Substituting these to equation (14), we obtain the following equation.

$$\left(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2\right) = \left(\begin{array}{ccc:ccc} 7 & 11 & 2 & 4 & 5 & 7 \\ 0 & 6 & 8 & 0 & 2 & 7 \\ 0 & 0 & 3 & 0 & 0 & 3 \end{array}\right) \left(\begin{array}{ccc:ccc} 7 & 0 & 0 & 4 & 2 & 1 \\ 0 & 17 & 0 & 0 & 5 & 6 \\ 0 & 0 & 13 & 0 & 0 & 10 \\ \hdashline 4 & 0 & 0 & 4 & 0 & 0 \\ 2 & 5 & 0 & 0 & 7 & 0 \\ 1 & 6 & 10 & 0 & 0 & 17 \end{array}\right)^{-1} \tag{21}$$

As we have seen before, we can confirm that

$\hat{\mathbf{E}}$ part in Eq.(16) is an upper triangular matrix and
$\hat{\mathbf{D}}, \hat{\mathbf{F}}$ part in Eq.(16) are diagonal matrices.
$\hat{\mathbf{E}}^T$ part is there by a lower triangular matrix.

We can find that if $\hat{\mathbf{E}}$ part becomes an upper triangular matrix, then the items compose upper shift or the same level shift. Calculation results of $\left(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2\right)$ become as

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follows.

$$\left(\hat{\mathbf{A}}_1, \quad \hat{\mathbf{A}}_2\right) = \begin{pmatrix} 0.979 & 0.621 & 0.091 & 0.021 & -0.009 & 0.081 \\ 0.113 & 0.446 & 0.776 & -0.113 & -0.065 & -0.208 \\ -0.091 & -0.067 & 0.133 & 0.116 & 0.074 & 0.127 \end{pmatrix} \quad (22)$$

One step forecasting can be obtained by Eq.(7) under the utilization of estimated parameters of Eq.(22). When making forecast by this method, "former buying data" and "before former buying data" are required as is stated at the beginning of 4.

7. Conclusion

Consumers often buy higher grade brand products as they are accustomed or bored to use current brand products they have.

Formerly we have presented the paper and matrix structure was clarified when brand selection was executed toward higher grade brand. Takeyasu and Higuchi (2007) suggested that matrix structure was analyzed for the case brand selection executed for upper class. In this paper, equation using transition matrix was extended to the second order lag and the method was newly re-built. One of the TRIZ methods was extended and applied. In this paper, the problem was to improve forecasting accuracy. Method was taken by building the expansion to the second order lag model in order to improve forecasting accuracy to the objective value. Detailed inspection was executed in the numerical example, matrix structure's hypothesis was verified. We can utilized the data as it is for the data in which time lag exist by this new model and estimation accuracy of parameter becomes more accurate and forecasting becomes more precise. Such research as questionnaire investigation of consumers' activity in automobile purchasing should be executed in the near future to verify obtained results.

References

Aker, D. A. (1991). *Management Brands Equity*. USA: Simon & Schuster.

Katahira, H. (1987). *Marketing Science* (In Japanese). Tokyo University Press.

Katahira, H. & Sugita, Y. (1994). Current movement of Marketing Science (In Japanese). *Operations Research*, *4*, 178-188.

Takahashi, Y. & Takahashi, T. (2002). Building Brand Selection Model Considering Consumers Royalty to Brand (In Japanese). *Japan Industrial Management Association*, *53*(5), 342-347.

Takeyasu, K. & Higuchi, Y. (2007). Analysis of the Preference Shift of Customer Brand Selection. *International Journal of Computational Science*, *1*(4), 371-394.

Yamanaka, H. (1982). Quantitative Research Concerning Advertising and Brand Shift (In Japanese), *Marketing Science*. Chikra-Shobo Publishing.

Zlotin, B. & Zusman, A. (2006). *Patterns of Evolution: Recent Finding on Structure and Origin*. TRIZCON 2006.

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